The Ring Problem and its connection to some known problems of Celestial Dynamics

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Abstract

The aim of this contribution is to show that some of the dynamic systems which have been presented in the past in the international bibliography, may be studied by means of a general model of N+1 bodies with a symmetric geometric configuration, the so called 'ring problem'.

1 Introduction

An ambitious target as well as a challenge for a researcher who works on the scientific area of many-body systems, is to find general reliable models that can be used in many particular cases. The international journals and the proceedings of many conferences often offered in the past extend hospitality to papers referring to various dynamic models. However most of them dealt with systems including a limited number of bodies, usually three, four or five. Moving on the direction of constructing general models with any number of bodies, we have proposed (Kalvouridis, 1999) a general model consisting of N+1 bodies which we have called the "ring problem" after its geometrical configuration. The great advantage of this model is that by simple changing its parameters, it can be reduced to some known particular problems. Following herewith we shall confine to the most relevant examples concerning the famous Copenhagen case of the restricted three-body problem, the restricted four-body problem of Marañhao, the restricted five-body problem of Ollongren and the Caledonian problem of Roy and Steves.

2 Dynamic properties of some known problems of Celestial Mechanics under the view of the ring problem

The ring problem is characterized by two parameters, namely the number ν of the peripheral primaries and the ratio $\beta = m_0/m$ of the mass of the central primary P_0 to the mass of a peripheral one. The motion of the particle in the physical space is described in a synodic coordinate system initiated at the mass center P_0 of the system by means of the differential equations

$$\begin{aligned} \ddot{x} - 2\dot{y} &= \frac{\partial U}{\partial x} \\ \ddot{y} + 2\dot{x} &= \frac{\partial U}{\partial y} \\ \ddot{z} &= \frac{\partial U}{\partial z} \end{aligned}$$

whereas

$$U(x, y, z) = \frac{1}{2}\left(x^2 + y^2\right) + \frac{1}{\Delta}\left[\frac{\beta}{r_0} + \sum_{i=1}^{\nu} \frac{1}{r_i}\right]$$

is the potential function, r_0 and r_i, i =1,..,ν are the distances of the particle from each individual primary, θ and ψ are the angles formed between the primaries, and

$$\Delta = M(\Lambda + \beta M^2)$$
$$\Lambda = \sum_{i=2}^{\nu} \frac{\sin^2\theta \cos(\frac{\nu}{2} + 1 - i)\theta}{\sin^2(\nu+1-i)\theta}$$

where M = $[2\,(1-\cos\psi)]^{1/2}$.

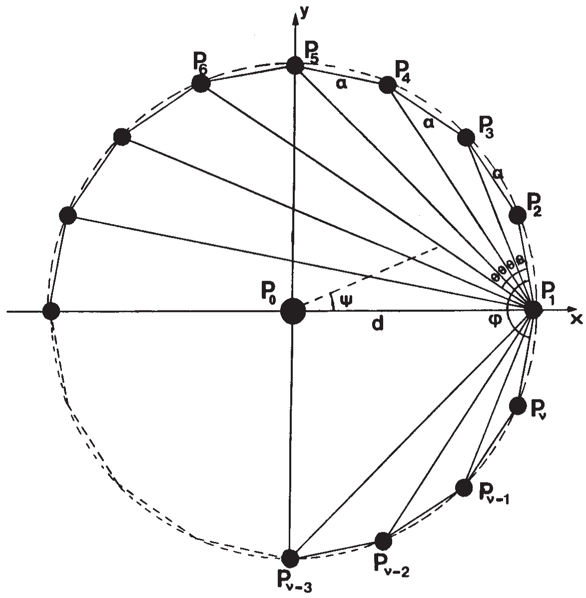


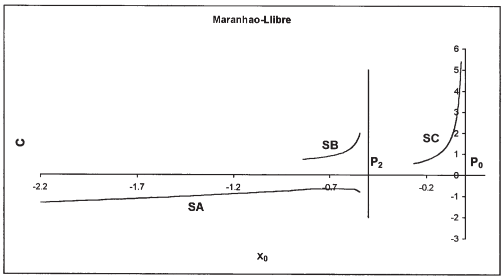
Figure 1: The general primarie's configuration of the ring problem

2.1 The Copenhagen case of the restricted three-body problem

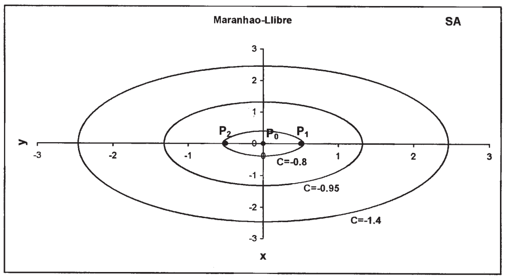
This special case of the restricted three body problem was widely studied during the first three decades of the 20^{th} century. Szebehely in his relevant book (Szebehely, 1967) refers to over 30 papers published in *Copenhagen Observatory Publications.* By using the ring model with ν=2 and β=0, we have verified the obtained in the past results by recalculating some of the families, as for example the classes f (Burrau and Strömgren, 1915), g (Burrau and Strömgren, 1915; Strömgren, 1922) and m (Möller, 1935).

2.2 The restricted four-body problem of Marañhao

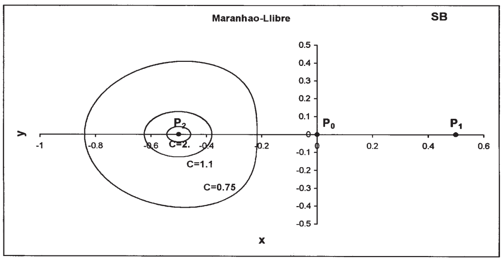
The problem describes the motion of a mass point of negligible mass under the Newtonian gravitational attraction of three bodies (called the primaries) moving in circular periodic orbits around their center of mass fixed at the origin of the coordinate system (Marañhao, 1995; Marañhao and Llibre, 1999). At any moment, the primaries form a collinear equilibrium configuration of the three-body problem. Two of these primaries have equal masses and are located symmetrically with respect to the third primary which has different mass and rests motionless at the mass center of the system. By using the general model of the ring problem with $\nu = 2$ and $\beta \neq 0$we are able to study the dynamic features of this system. As an example we give the characteristic curves and some members of three families of simple periodic orbits found so far (Figure 2). The orbits of the first family SA encircle all three primaries. The orbits of SB surround the peripheral primary P_2 and the orbits of SC encounter the central primary P_0.



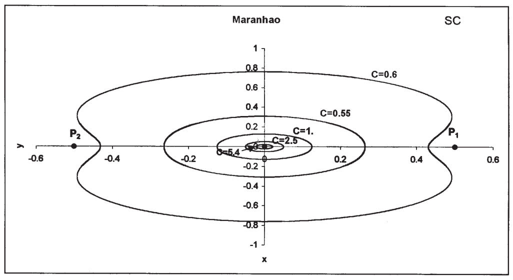
(a) Characteristics in the (x_0,C) space.



(b) Family SA.



(c) Family SB.



(d) Family SC.

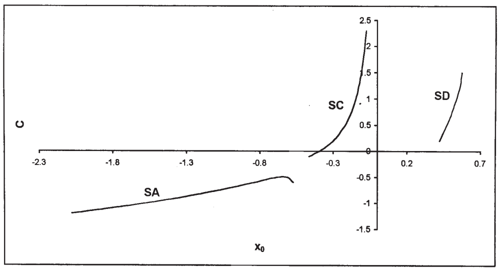
Figure 2: Families of simple periodic orbits in the restricted four-body problem of Marañhao (ν=2, $\beta \neq$0).(a) Initial conditions. (b)- (d). Evolution of the orbits belonging to the families SA, SB and SC

2.3 The restricted five-body problem of Ollongren

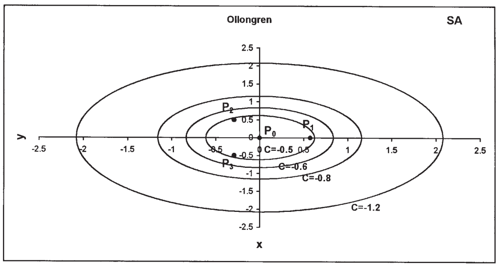
Three bodies of equal mass m, are located at the vertices of an equilateral triangle. Another body of mass β times m is located at the mass center of the system. The three peripheral primaries revolve around their mass centre in circular orbits under the influence of their mutual gravitational attraction. A fifth body of negligible mass compared to m, moves in the plane of the primaries under their gravitational attraction. The problem deals with the small body's motion (Ollongren, 1988) and can be treated by using the ring model with ν=3 and $\beta \neq$0. Figure 2 shows the evolution of the characteristic curves in the initial conditions space (x_0, C), as well as some orbits- members of three families of simple periodic orbits, namely SA, SC and SD, where we have considered that β=50. Family SA consists of planetary type orbits which encircle all four primaries. Family SC consists of orbits surrounding the central primary P_0, while the orbits of family SD are described around the peripheral primary P_1.

2.4 The Caledonian problem

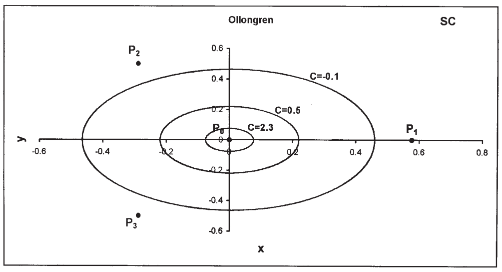
The Caledonian problem formulated by Steves and Roy (1998), refers to a configuration which involve four primaries of equal mass which move in coplanar orbits about the centre of mass of the system. The most interesting types of geometric four-body equal mass arrangements are the square and the triangle. The particle's motion in such a configuration, can be studied by means of the ring model with ν=4 and β=0, or ν=3 and β=1. Figure 4 shows the characteristics and the evolution of the simple periodic orbits of three families found during the study of the first case.



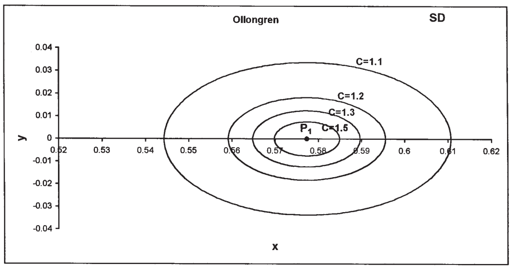
(a) Characteristics in the (x_0,C) space.



(b) Family SA.

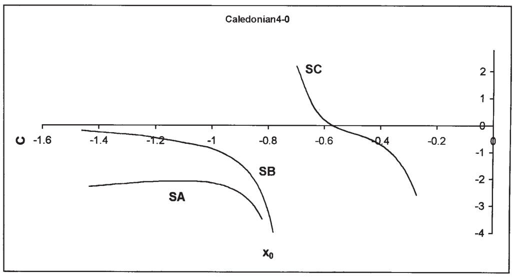


(c) Family SC.

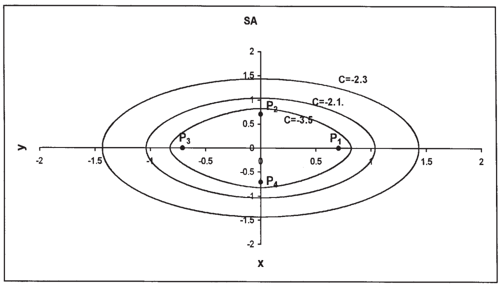


(d) Family SD.

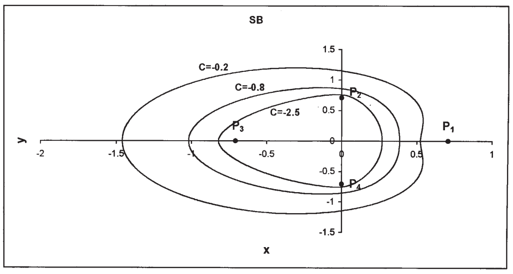
Figure 3: Initial conditions and families of periodic orbits of the Ollongren's problem



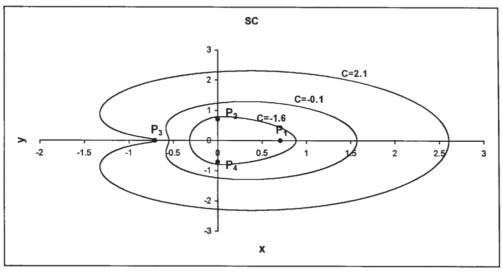
(a) Characteristics in the (x_0,C) space.



(b) Family SA



(c) Family SB.



(d) Family SC.

Figure 4: Initial conditions and families of periodic orbits in the square equal mass configuration of the Caledonian problem (ν=4, β=0)

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