Relaxation time of N-body systems.

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Abstract

We study the relaxation time of an N-body configuration, by measuring the angular deflection of test particles launched in a rigid configuration of field particles. We find that relaxation time depends on the central concentration of the configuration. Highly centrally concentrated systems have a shorter relaxation time than configurations with a homogeneous distribution. For the latter we confirm that relaxation time is proportional to the number N of particles in the distribution. On the other hand, the relaxation time of centrally concentrated systems has a much shallower dependence on N. We also find that relaxation time increases with the inter-particle velocity and with the softening used in simulations. The dependence on the softening is not very strong and a change of one order of magnitude in the softening is needed to double the relaxation time.

1 Introduction

An N-body system is usually assumed to represent a smooth, continuous, density profile that extends over all the space that the system occupies. Similarly we assume that the stars (the particles or bodies) of the system move in the smooth potential this density produces. This theoretical approach is generally valid if the number of bodies N of the system tends to infinity.

In real systems, where N is finite, the mass is confined to the stars. In other words the real density distribution is not smooth and continuous but discrete. Thus, the orbit of a star is not simply due to a smooth theoretical potential. Two-body encounters produce local perturbations in the potential and this results to a divergence of the real orbit from the theoretical one. This effect increases the velocity dispersion of the stars with time and drives the system to a state of higher entropy.

For galaxies, due to the very big number of stars, this effect is very small in a time-scale comparable to their life-time. But for star clusters and for N-body simulations, where N is small due to the limitations induced by the speed of computers, relaxation effects may become important. The number of particles (currently) used in a computer simulation is several orders of magnitude less than the number of stars in a typical galaxy. Therefore, relaxation effects may become important and lead the simulations to incorrect results.

The characteristic time that is linked to the relaxation effect is called relaxation time. It can be defined as the time needed for the energy or the velocity of a particle to change by an amount of the order of its initial energy or velocity. In other words, if v is the initial velocity of a star and Δv is the change in the velocity in one crossing of the system T_{cross}, then the relaxation time T_{relax} is

$$T_{\mathrm{relax}} = \frac{v^2}{\Delta v^2} T_{\mathrm{cross}} \tag{1}$$

Analytic estimates of the relaxation time have already been given in the literature. A simple estimate (Binney and Tremaine 1995), assuming a homogeneous distribution and virial equilibrium ($v^2 \approx \frac{GNm}{R}$), gives

$$T_{\rm relax} = \frac{N}{8 \ \ln N} T_{\rm cross}. \tag{2}$$

In the case of computer simulations, where a softening parameter ϵ is used in the potential, an analysis similar to that of Binney & Tremaine leads to

$$T_{\rm relax} \quad = \quad \frac{v^4 R^2}{G^2 M^2} \frac{N}{8 \ln(R/\epsilon)} T_{\rm cross} = \frac{N}{8 \ln(R/\epsilon)} T_{\rm cross} \tag{3}$$

The above equations give an estimation of the relaxation time but suffer from one major drawback. They have been derived assuming a homogeneous distribution of stars! But not all real N-body system are homogeneous. Does the density distribution in the system play any role? For real system, that are centrally concentrated, can we trust the results of the above equations or is their relaxation time much sorter?

2 The method and numerical miscalenea

Let us consider a sphere of a given density profile represented by N fixed particles, hereafter called field particles, and let us place a test particle of zero mass at the edge of this sphere with a radial velocity $v = f_{\rm v} \ v_{\rm esc}$, where $v_{\rm esc}$ the escape velocity and $f_{\rm v}$ a multiplicative constant. In the limit of $N \rightarrow \infty$ its trajectory will be a straight line passing through the center of the sphere. For a finite N, however, the test particle will be deflected by a number of encounters with the field particles and thus it will cross the surface of the sphere at an angle Φ from the corresponding theoretical point. Following Standish & Aksnes (1969), we can measure the relaxation time as

$$T_{\rm relax} = \frac{T_{\rm cross}}{sin^2 \Phi}, \tag{4}$$

where $T_{\rm cross}$ is the crossing or transit time.

Different realizations of the adopted model will of course give somewhat different relaxation times. It is thus better to use an estimate based on the average of many realizations or trajectories. Thus we have

$$T_{\rm relax} = \frac{< T_{\rm cross} >}{< sin^2 \Phi >}, \tag{5}$$

where the $<>$ denote an average over all realizations and/or all trajectories.

To find the effect of central concentration on the relaxation time we use three different mass distributions, namely the homogeneous sphere (model H), the Plummer model (model P) and the Dehnen sphere (model D). For the Plummer model the density is given by

$$\rho(r) = \frac{3 M_T}{4 \pi a_P^3} (1 + r^2/a_P^2)^{-5/2} \tag{6}$$

while for the Dehnen model (Dehnen 1993) by

$$\rho(r) = \frac{(3-\gamma) M_T}{4 \pi} \frac{a_D}{r^\gamma (r + a_D)^{(4-\gamma)}}, \tag{7}$$

where M_T is the total mass of the system, a_P and a_D the scale-lengths of model P and model D respectively and γ is the concentration index of Dehnen model. In the following numerical application we have adopted for the scale-lengths of the two models the values $a_P = 9.2$ and $a_D = 3.1$ respectively, in order to have the

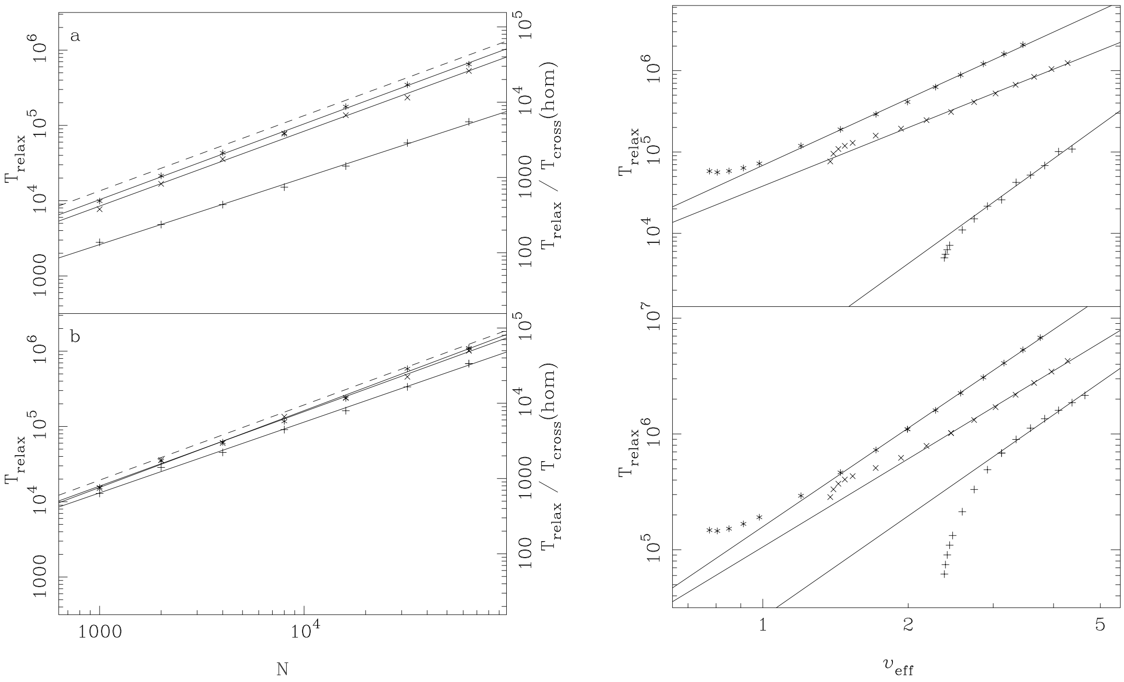

Figure 1: The dependence of the relaxation time on the number of particles (left) and on the particle velocity (right) for model H (stars), model P ($\times$) and model D (crosses). Upper panels are for $\epsilon = 0.01$, while lower are for $\epsilon = 0.5$.

75% of the total mass within the same radius $R = 20$. This radius is considered as a cutoff radius for all three models. The total mass of all models within the cutoff radius was taken equal to 15. The concentration index γ was taken equal to 1,

For each model we take 10 different field particle distributions and for each we calculate 1 000 test particles trajectories. For each of the test particles we calculate the crossing time T_{cross} and the deflection angle from the theoretical (straight line) orbit, Φ. Then the relaxation time is calculated using equation (5).

The forces between particles were calculated on the Marseille's GRAPE-3AF and GRAPE-4 systems using direct summation. This of course limits us to Plummer softenings, but allows us, because of the speed of GRAPE, to explore a range of values for all parameters. *(To CV : since now we have space we could add references to GRAPE3 and G4, as in the paper)*

3 Results and Conclusions

We find that the relaxation time increases with the softening, but that this dependence is not too strong. The effect of changing the softening by an order of magnitude is simply an increase of the relaxation time by a factor of two. Softening, however, can not be used to control the relaxation time, because for too much softening the forces will be quite far from Newtonian. For each configuration of an N-body system there is an optimum value of softening which makes the resulting forces more close to the theoretical ones (Merritt 1996, Athanassoula et al. 2000).

On the other hand, the dependence of the relaxation time on the number N of particles is stronger. In the left part of Figure 1 we see the relaxation time as a function of N for our three models, namely model H (stars), model P ($\times$) and model D (crosses). The upper panel is for $\epsilon = 0.01$ while the lower is for $\epsilon = 0.5$. The ϵ in the upper panel is near the optimal for model D (≈ 0.04), while in the lower it is near the optimal for the other two models. Solid straight lines are least square fits, while the dashed line is the prediction of eq. (3).

As we can see there is a power law dependence of T_{relax} on N. Model H shows a behavior well represented by the analytical estimates of eq. (3). The other models, P and D, have a lower relaxation time. The more centrally concentrated the model is, the less is the value of $T_{\rm relax}$. Also note that the dependence of $T_{\rm relax}$ on N changes from linear, in model H, to a more shallow one, as the central concentration increases. Thus, when we increase the number of particles in the configuration, we increase the relaxation time relatively less in more centrally concentrated configurations than in less centrally concentrated ones. In other words not only is the relaxation time smaller in the more centrally concentrated configurations, but also it takes more particles to increase it by a given amount.

The dependence of the relaxation time on the particle velocity is given in the right part of Figure 1. The quantity $v_{\rm eff}$ in the abscissa is a measure of the velocity of encounters between field and test particles. Again the upper panel is for $\epsilon = 0.01$ and the lower one for $\epsilon = 0.5$. The dependence is linear in the log-log plane for large values of the effective velocity and deviates strongly from linear for smaller values. Note the very strong drop of the relaxation time for model D at low $v_{\rm eff}$ values. This should be due to the fact that, for low initial velocities, the orbit deviates strongly from the theoretical straight line trajectory. The central concentration further enhances this deviation, resulting in a much lower relaxation time.

In conclusion we can say that relaxation effects are not important in simulations of violent events like galaxy collisions and merging, but may play a role in quasi-equilibrium systems. The effect is more important if the number of particles in the system is relatively small and especially if the system is centrally concentrated. In this case the relaxation time may be two or more orders of magnitude smaller that the theoretically predicted. Systems like globular clusters may have relaxation times much smaller than their age and this may be an explanation, apart from the formation process, of why these systems appear to be spherical.

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