The turn-over scenario for white dwarfs and neutron stars

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Abstract

We study numerically the so-called "turn-over scenario" for rotating magnetic white dwarfs and neutron stars. According to this scenario, the magnetic symmetry axis of the model inclines at a gradually increasing angle (the so-called "turn-over angle") relative to the invariant angular momentum axis. Consequently, the model becomes "perpendicular rotator" (i.e., its turn-over angle becomes almost 90^o) on a "turn-over timescale" calculated to be $\sim 10^7$ yr for typical white dwarf models and $\sim 10^1 - 10^3$ yr for typical neutron star models. Furthermore, the initial differential rotation of the model turns to uniform rotation due to angular momentum mixing caused by hydrodynamic Alfven waves propagating along the poloidal magnetic field lines. Our numerical results show that, during the turn-over phase, the spin-down time rate is large, while the spin-down power remains small; so, the turn-over phase is a characteristic case for which an eventually observed large spin-down time rate should not be interpreted as implying a large spin-down power.

1 Introduction

"Turn-over" or "turn-over phase" or "turn-over process" is called the gradual increase of the angle between the magnetic symmetry axis and the rotation axis of a star from $\sim 0^o$ (axisymmetric or aligned rotator) to $\sim 90^o$ (orthogonal or perpendicular rotator). Any intermediate configuration is called "oblique rotator". The angle χ between the magnetic and the rotation axis is called "turn-over angle" or "obliquity angle". We say that we study a "turn-over scenario" if we take into account the turn-over phase in this scenario.

Actually, during the turn-over phase, it is the angular momentum that remains invariant (its axis is taken to be the z axis); while the angular velocity is inclined at a small angle γ (less than 1^o, say) with respect to the angular momentum axis. At a first approximation and without loss of generality, we can assume that these two axes coincide.

Turn-over seems to be a rather underestimated phenomenon. Indeed, searching the NASA Astrophysics Data System (`http://adswww.harvard.edu`), we find that, during the last 30 years, $\sim$ 12000 papers have been published with the keywords `(white AND dwarf) OR (neutron AND star) OR (pulsar)` in their title; while, at the same time, roughly 100 papers have in their title the keywords `(oblique AND rotator) OR (perpendicular AND rotator)`.

This is strange enough, since turn-over is closely related to the well-known lighthouse model: as the pulsar spins on its axis, the emission beam (with symmetry axis coinciding with the magnetic symmetry axis of the star) sweeps the space, crossing periodically our line of sight. This happens only if $\chi > 0$, since otherwise (that is, for an "aligned rotator") we can observe just a radiation source coinciding with one of the magnetic poles of the emitting star; so, an aligned rotator lacks the most significant characteristic of a typical pulsar.

In the turn-over scenario, the casting of the roles has mainly to do with rotation, toroidal field, poloidal field, and turbulent viscosity. In particular, rotation (1) builds up the most efficient energy reservoir, i.e., the rotational kinetic energy, which then powers on a variety of interesting phenomena; and (2) induces oblate configurations due to centrifugal forces.

The poloidal magnetic field (1) turns on the magnetic dipole mechanism, which then converts rotational kinetic energy into radiated energy; and (2) cooperates with rotation in inducing oblate configurations, i.e., both these partners tend to derive oblate configurations.

The toroidal magnetic field (1) stabilizes the poloidal magnetic field structure; and (2) tends to derive prolate configurations, that is, it has opposite action to both rotation & poloidal field.

Finally, the turbulent viscosity causes drastic energy dissipation and, thus, brings the configuration from an unstable high energy level (aligned rotator) to a stable low energy level (perpendicular rotator).

2 The turn-over scenario

The reader can find a full discussion on this subject in Geroyannis 2001 (hereafter G01; symbols and definitions used here are identical to those in G01). A brief description of the turn-over scenario has as follows.

Early evolutionary phase (EEP)

(1) We assume an axisymmetric differentially rotating magnetic model, undergoing an axisymmetric early evolutionary phase of secular timescale, t_{SEC}, during which rotation & poloidal field dominate over the toroidal field and derive oblate configurations with moment of inertia I_{33} (where the principal axis I_3 coincides with the spin axis) greater than $I_{11}(= I_{22})$; the other two principal axes I_1 and I_2 lie on the equatorial plane.

(2) During the EEP, the model suffers from secular angular momentum loss and spin-down due to the magnetic dipole mechanism, activated by the poloidal field; this leads to secular decrease of both ellipticity and I_{33}.

Late evolutionary phase (LEP)

(1) At some particular time signaling the beginning of a late evolutionary phase, the toroidal field starts prevailing over rotation & poloidal field, first succeeding in equating the moments of inertia, $I_{11} = I_{33}$, and then in establishing dynamical asymmetry, $I_{11} > I_{33}$.

(2) "Dynamically asymmetric configurations" (DAC) tend to turn over spontaneously, so that to rotate about axis with moment of inertia greater than I_{33}, with the angular momentum remaining invariant; thus, the fate of a DAC is to become oblique rotator and, eventually, perpendicular rotator.

Turn-over phase (TOP)

(1) During the turn-over phase, the turn-over angle, χ, increases spontaneously up to $\sim 90^o$ on a turn-over timescale, t_{TOV}. The terminal model rotates about its I_1 axis, coinciding with the invariant angular momentum axis, and occupies the state of least energy consistent with its angular momentum and magnetic field.

(2) The excess energy due to differential rotation, defined by the angular velocity component Ω_3 along the spontaneously turning over I_3 axis, coinciding in turn with the magnetic symmetry axis, is dissipated down to zero due to the very efficient action of turbulent viscosity in the convective zone of the model. So, the terminal model does not rotate about its magnetic symmetry axis.

(3) It seems very difficult for the terminal model to sustain differential rotation along its I_1 axis, mainly due to the destructive action of the poloidal field. There is a competition between the efforts of the magnetic stresses to remove rotational nonuniformities, and those of the rotational velocities to bury and destroy the magnetic flux. If the magnetic field and the electrical conductivity have appropriate values, then the magnetic field prevails and removes all the nonuniformities of rotation. So, the terminal model rotates rigidly about its I_1 principal axis with angular velocity Ω_1.

A critical question is *if the magnetic field is strong enough to remove differential rotation.* This interesting problem is discussed in full detail in G01 (§ 7); the discussion reveals a critical issue concerning the starting model. In particular, it seems that the toroidal magnetic field must be strong enough so as to induce an adequate dynamical asymmetry for the turn-over to take place; while, at the same time, it must be weak enough so as to permit interchanging of angular momentum via hydromagnetic Alfvén waves propagating across its field lines. The difficulty is that, on the one hand, dynamical asymmetry increases with the toroidal magnetic field and, on the other hand, efficiency in interchanging angular momentum decreases as the toroidal field increases. In view of this remark and to the extent that our problem concerns mainly the behavior of the starting model, the right question seems to be: *is the toroidal magnetic magnetic field strong enough to induce dynamical asymmetry and weak enough to permit angular momentum mixing?.*

An approximative estimate for the turn-over timescale, t_{TOV}, is given by (Mestel & Takhar 1972, hereafter MT72, eq. [54]; see also some interesting remarks regarding differential rotation at the last two paragraphs of § 3)

$$t_{TOV} \simeq \frac{DE_{DRD}}{V} \left\langle \Omega_3 \right\rangle^{-2} \left\langle \frac{H^2}{8\pi \varrho a^2} \right\rangle^{-2} \left\langle \frac{1}{3} \, \frac{\varrho V_t}{\Lambda_t} \right\rangle_{sz}^{-1} \left\langle \frac{\Omega_3^2 r^2}{a^2} \right\rangle_{sz}^{-2} \left\langle \Lambda_t \right\rangle_{sz}^{-2} , \qquad (1)$$

where the subscript sz denotes that the corresponding average is calculated over the surface zone with base at the transition layer, ξ_s, and top at the boundary, ξ_1, of the star; averages without subscripts are calculated over the whole star, i.e., they are global averages. This estimate results as the mean energy per unit volume available for dissipation, DE_{DRD}/V (DE_{DRD} is the excess rotational kinetic energy due to differential rotation and V is the volume of the configuration), divided by the mean energy dissipation during the turn-over due to turbulent viscosity in the convective surface zone per unit volume and per unit time (i.e., the product of the last 5 terms in the right-hand side of eq. [1]; details on the derivation of such a relation are given in MT72, eqs. [7], [49]–[50]).

3 Some indicative results

Table 1 gives some significant quantities involved in the turn-over scenario. Our computations concern two models of compact stars. The first model simulates the AE Aqr white dwarf; the corresponding computations have been performed by the so-called "complex plane iterative technique" (CIT; Geroyannis & Papasotiriou 2000, and references therein). The second model simulates the 1987A neutron star; the corresponding computations should be considered as rough computations. An accurate numerical treatment of neutron stars is now in preparation.

The results for the AE Aqr white dwarf can simplify the study of the well-known spin-down problem in this white dwarf. In particular, during the turn-over phase, the observed spin-down time rate (interpreted as spin-down time rate due to turn-over) is large, $\dot{P} \simeq 5 \times 10^{-14}\,\mathrm{s\,s^{-1}}$; while the estimated spin-down power, $\dot{T} \simeq 5 \times 10^{32}\,\mathrm{erg\,s^{-1}}$, is almost two orders of magnitude less than the classically estimated spin-down power (for details, see G01, § 1). So, an interesting conclusion is that: *an observed large spin-down time rate does not always imply a large spin-down power.*

Our results for the 1987A neutron star show that it needs $t_{SEC} \simeq 8.5\,\mathrm{yr}$ to complete its early enolutionary phase, and $t_{now} \simeq 8.5\,\mathrm{yr}$ to obtain an observationally favourable turn-over angle (in accordance with the lighthouse model), $\chi \simeq 63^o$; the full turn-over timescale for this star is $t_{TOV} \simeq 16\,\mathrm{yr}$. In conclusion, *we expect that we will be able to observe the 1987A neutron star as typical pulsar within the period 2004 (i.e.,* $1987 + 8.5 + 8.5\,\mathrm{yr}$*) – 2012 (i.e.,* $1987 + 8.5 + 16\,\mathrm{yr}$*).*

Table 1: Summary of calculations for the AE Aqr white dwarf (accurate computations) and the 1987A neutron star (rough computations). Parameters in cgs units, unless stated otherwise. Parenthesized numbers denote powers of 10. Basic model parameters: $F_r = 1.00$; $h = 0.08$ (symbols and definitions used here are identical to those used in G01)

Parameter	AE Aqr WD	1987A NS
Mass, M (solar masses)	8.9(−01)	1.6(+00)
Initial turn-over angle, χ_{ini} (arcdegrees)	1.4(+00)	1.3(+00)
Angular momentum, L_{xx}	2.2(+49)	2.3(+48)
Average surface poloidal field, B_s	2.9(+06)	3.0(+13)
TOV timescale, t_{TOV} (yr)	9.0(+06)	1.6(+01)
Secular timescale, t_{SEC} (yr)	5.0(+08)	8.5(+00)
Present TOV time, t_{now} (yr)	4.6(+06)	8.5(+00)
Present central period, P_{now}	3.31(+01)	2.14(−03)
Present turn-over angle, χ_{now} (arcdegrees)	7.2(+01)	6.3(+01)
Average spin-down time rate due to turn-over, $\left\langle \dot{P} \right\rangle_t$	5.6(−14)	1.7(−12)
Average power loss due to turn-over, $\left\langle \dot{T} \right\rangle_t$	4.9(+32)	4.1(+41)

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References

Geroyannis, V. S. 2001 astro-ph/0103080 (G01)
Geroyannis, V. S., & Papasotiriou, P. J. 2000 ApJ, 534, 359
Mestel, L., & Takhar, H. S. 1972 MNRAS, 156, 419 (MT72)