Viscous dissipative effects in white dwarfs and neutron stars

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Abstract

When a differentially rotating magnetic star undergoes "turn-over", i.e., its magnetic symmetry axis is inclining at a gradually increasing angle with respect to its invariant angular momentum axis up to the perpendicular position (situation known as "perpendicular rotator"), the poloidal magnetic field causes interchange of angular momentum via hydromagnetic Alfven waves propagating along the poloidal magnetic field lines and, accordingly, the resulting perpendicular rotator obtains uniform rotation. This dissipative mechanism is represented by the so-called "turn-over viscosity". For several white dwarf and neutron star models, we compute the energy dissipation due to the turn-over viscosity; we also compare this effect with the corresponding dissipative effects induced by the electron, ion, and turbulent viscosity.

1 Introduction

In the so-called "turn-over scenario" (for details see Geroyannis 2001, hereafter G01, § 3; symbols and definitions used here are identical to those used in G01), we develop a phenomenological model involving time evolution of the turn-over process (G01, § 6). In detail, in the so-called "mixer-mixture model" (MMM) we assume that the turning over configuration acts as a "mixer" on the "mixture" configuration, which remains almost axisymmetric with respect to the invariant angular momentum axis, i.e., the z axis (the meaning of "almost axisymmetric configuration" is that the angle γ between its angular velocity and its invariant angular momentum remains small: $\gamma < 1^o$, say; apparently, γ becomes zero after the termination of the turn-over; in the following, we assume that such a deviation of the mixture configuration from axisymmetry is negligible and we treat it as being axisymmetric with respect to the z axis).

The mixer consumes its excess rotational kinetic energy due to differential rotation in the process of homogenizing the rotation of the mixture. In particular, when viewed in the mixture's system of reference, the mixer generates a velocity field, which carries mass elements across cylinders of different angular velocities and momenta. This is due to the fact that the rotational circular orbit of a mass element of the mixer, as seen in the mixer's system of reference, crosses several cylinders with different angular velocities, as seen in the mixture's system of reference. Thus, there is an interchange of angular momentum, which leads to uniform rotation of the mixture.

2 Dissipation of energy due to turn-over

The so-called turn-over viscosity, μ_{TOV}, can be written as (G01, eq. [58])

$$\mu_{TOV}(\xi, \nu, t) \simeq \frac{1}{3}\varrho(\xi) A^v_{TOV} V_{TOV}(\xi, \nu, t) A^h_{TOV} h_P(\xi)\,. \tag{1}$$

Then the energy, DE_{TOV}, dissipated over the time t_{TOV} due to turn-over is given by (G01, eq. [59])

$$\begin{aligned} DE_{TOV} &= \int_0^{t_{TOV}} D_{TOV}(t)\, dt = 4\pi\alpha^3 \times \\ & \int_0^{t_{TOV}} \Omega_{3c}^2(t) \left(\int_0^1 \int_0^{\xi_1} \mu_{TOV}(\xi,\nu,t)\, \xi^4 (1-\nu^2) \left(\frac{\partial \omega(s, F_r(t))}{\partial s} \right)^2 d\xi d\nu \right) dt \end{aligned} \tag{2}$$

where $D_{TOV}(t)$ is the power dissipated due to turn-over. On the other hand the energy, DE_t, dissipated over the time t_{TOV} due to turbulent viscosity (the latter acting over a "turbulent zone" $[\xi_t, \xi_1]$ lying, in turn, within the surface zone $[\xi_s, \xi_1]$), is given by

$$\begin{aligned} DE_t &= \int_0^{t_{TOV}} D_t(t)\, dt = 4\pi\alpha^3 \times \\ & \int_0^{t_{TOV}} \Omega_{3c}^2(t) \left(\int_0^1 \int_{\xi_t}^{\xi_1} \mu_t(\xi,\nu,t)\, \xi^4 (1-\nu^2) \left(\frac{\partial \omega(s, F_r(t))}{\partial s} \right)^2 d\xi d\nu \right) dt\,, \end{aligned} \tag{3}$$

where $D_t(t)$ is the power dissipated due to turbulent viscosity.

Consequently, the required equality $DE_{TOV} = DE_t$ can be used for accurately determining the base ξ_t of the turbulent zone. In the following section, we shall use this method for several white dwarf and neutron star models. Actually, we shall determine the so-called "representative turbulent zone" (RTZ, with $\xi_t = \xi_{RTZ}$), which corresponds to an axisymmetric model with constant differential rotation, $F_r = F_{r[ini]}$, and angular velocity, $\Omega_{3c} = \Omega_{3c[ini]}$, suffering from power loss due to turbulent viscosity, D_t, equal to the average value (over the time t_{TOV}) of the power loss due to turn-over, $\langle D_{TOV} \rangle_t$; that is,

$$4\pi\alpha^3 \Omega_{3c[ini]}^2 \int_0^1 \int_{\xi_{RTZ}}^{\xi_1} \mu_t(\xi,\nu,t)\, \xi^4 (1-\nu^2) \left(\frac{\partial \omega(s, F_{r[ini]})}{\partial s} \right)^2_{s=s(\xi,\nu)} d\xi d\nu = \langle D_{TOV} \rangle_t \,. \tag{4}$$

In order for this method to be reliable, however, the power dissipated due to electron and ion viscosity of degenerate matter (Durisen 1973, § 3, eqs. [4], [6], respectively), D_{EIV}, should be small in comparison with the former, $D_t \gg D_{EIV}$; otherwise, the latter cannot be screened by the former and thus becomes significant in the framework of TOV scenario.

3 Some results

In this paper, we present some preliminary results. In particular, Table 1 gives the most interesting quantities involved in the subject discussed above. Our computations concern four models of compact stars. Two of them simulate white dwarfs; the corresponding computations have been made by the so-called "complex plane iterative technique" (CIT; Geroyannis & Papasotiriou 2000, and references therein). The other two models represent neutron stars; the corresponding computations should be considered as rough computations. An accurate numerical treatment of neutron stars is now in preparation.

As a summary, it seems that the representative turbulent zone is $\sim 1\%$ of the corresponding surface zone of these models; this fact shows the high efficiency of turbulent viscosity in dissipating the rotational kinetic energy due to differential rotation and, thus, leading the configuration from an unstable high energy level (aligned rotator) to a stable low energy level (perpendicular rotator) within a relatively small turn-over timescale.

Table 1: Summary of calculations regarding four indicative models. Parameters in cgs units, unless stated otherwise. Parenthesized numbers denote powers of 10. WD: white dwarf model (Chandrasekhar's model), accurate computations. NS: neutron star model (Ideal Neutron Gas equation of state), rough computations. Basic model parameters: $F_r = 1.00$; $h = 0.08$. SZ: surface zone. RTZ: representative turbulent zone (symbols and definitions used here are identical to those used in G01)

Parameter	WD	WD	NS	NS
Mass, M (solar masses)	6.00(−01)	8.90(−01)	1.61(+00)	2.39(+00)
Delta parameter, δ (dimensionless)	1.00(+00)	4.41(+00)	1.00(+00)	1.00(+00)
Initial turn-over angle, χ_{ini} (arcdegrees)	1.41(+00)	1.41(+00)	1.32(+00)	1.41(+00)
Angular momentum, L_{xx}	7.00(+48)	2.17(+49)	2.31(+48)	2.00(+48)
Average surface poloidal field, B_s	5.00(+06)	5.00(+06)	3.00(+13)	2.00(+13)
Magnetic flux, f	4.01(+24)	2.28(+24)	4.44(+25)	1.90(+25)
TOV timescale, t_{TOV} (yr)	2.05(+07)	3.00(+06)	1.60(+01)	9.81(+02)
Present TOV time, t_{now} (yr)	1.36(+07)	1.56(+06)	8.52(+00)	8.55(+02)
Present central period, P_{now}	1.42(+02)	3.31(+01)	2.14(−03)	2.55(−03)
Average spin-down time rate due to turn-over, $\left\langle \dot{P} \right\rangle_t$	8.90(−14)	1.68(−13)	1.67(−12)	3.07(−14)
Present turn-over angle, χ_{now} (arcdegrees)	7.11(+01)	7.21(+01)	6.30(+01)	8.31(+01)
Average power loss due to turn-over, $\langle D_{TOV} \rangle_t$	1.71(+31)	1.45(+33)	4.06(+41)	6.19(+39)
Power loss due to nonturbulent viscosity, D_{EIV}	8.42(+28)	3.65(+30)	6.50(+38)	4.16(+38)
Percent SZ thickness, $\frac{\xi_1-\xi_s}{\xi_1} \times 100$	4.76(+00)	2.24(+00)	4.27(−01)	2.30(−01)
Percent RTZ thickness relative to SZ, $\frac{\xi_1-\xi_{RTZ}}{\xi_1-\xi_s} \times 100$	5.32(−01)	2.15(+00)	5.47(+00)	2.52(+00)
Central period, P_{xx}	1.04(+02)	2.48(+01)	1.69(−03)	1.72(−03)
Rotational kinetic energy, T_{xx}	1.47(+47)	1.81(+48)	3.06(+51)	2.55(+51)
Moment of inertia along the I_1 axis, $I_{DR,11}(h, L_{xx})$	1.79(+50)	1.39(+50)	9.15(+44)	8.45(+44)
Moment of inertia along the I_3 axis, $I_{DR,33}(h, L_{xx})$	1.74(+50)	1.38(+50)	9.05(+44)	8.23(+44)
Average surface poloidal field, B_s	5.00(+06)	5.00(+06)	3.00(+13)	2.00(+13)
Period, P_{RR}	1.62(+02)	4.07(+01)	2.54(−03)	2.67(−03)
Rotational kinetic energy, T_{RR}	1.36(+47)	1.67(+48)	2.86(+51)	2.35(+51)
Moment of inertia along the I_1 axis, $I_{RR,11}(h, L_{xx})$	1.80(+50)	1.41(+50)	9.31(+44)	8.49(+44)
Moment of inertia along the I_3 axis, $I_{33}(h)$	1.72(+50)	1.28(+50)	8.63(+44)	8.11(+44)
Average surface poloidal field, B_s	5.05(+06)	5.27(+06)	3.10(+13)	2.02(+13)

Acknowledgments

The research reported here was supported by the Research Committee of the University of Patras (C. Carathéodory's Research Project 1998/1932).

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