A numerical method for nonrelativistic computations of rotating magnetic compact stars obeying a variety of equations of state

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Abstract

We develop a time-saving iterative numerical method by combining (1) a modified–improved scheme of the so-called "piecewise polytropic approximation" (PPA), and (2) the so-called "complex-plane iterative technique" (CIT). We apply this method to nonrelativistic computations of constant-mass sequences of models with central densities intermediate to white dwarf and neutron star densities. This method proves to be particularly useful for problems involving heavy physical input (e.g., nonaxisymmetric configurations, plus differential rotation, plus toroidal and poloidal magnetic fields, plus electron, ion, turbulent, and turn-over viscosity, plus ...), because then a pure relativistic treatment becomes difficult and time-consuming.

1 Introduction

CIT has been first developed and used for computing rotating "visco-polytropic models" (Geroyannis 1991, hereafter Paper I, § 3); this term indicates that the differential rotation of such models results from the particular equations governing the viscosity of their material. This method has been then extended for computing rotating white dwarfs (Geroyannis & Papasotiriou 1994, hereafter Paper II), and rotating white dwarfs with toroidal magnetic field (Geroyannis & Papasotiriou 1997, hereafter Paper III). Recently (Geroyannis & Papasotiriou 1999, hereafter Paper IV), CIT has been further extended for computing rotating white dwarfs distorted by both toroidal and poloidal magnetic field(s). Finally, CIT has been used for computing constant-mass sequences of rotating magnetic white dwarfs and studying their spin-up and/or spin-down properties (Geroyannis & Papasotiriou 2000, hereafter Paper V).

PPA (Eriguchi & Müller 1985, hereafter EM85, § 2; Müller & Eriguchi 1985, hereafter ME85, § 2) has been first developed and used for computing equilibrium models of differentially rotating completely catalyzed zero-temperature configurations with central densities intermediate to white dwarfs and neutron star densities. Since then, the method has been used in most (mainly relativistic) computations of rotating compact stars as an inseparable part of the overall algorithms (see, e.g., Eriguchi, Hachisu, & Nomoto 1994, § 2.4). Actually, PPA is an almost unique choice when undertaking structure computations concerning several equations of state (see, e.g., Arnett & Bowers 1977, hereafter AB77).

In the present paper, we shall develop an iterative numerical method by combining both CIT and PPA; for brevity, we shall denote this method by C&P. The primary modification concerning PPA and involved in C&P is that, instead of determining a complete set of integration constants (as in ME85, eq. [5], and EM85, eq. [16]), we solve for a unique integration constant. Accordingly, the method is endowed by a "degree of freedom", which can be then used for "rescaling the units" and so computing a "starting model" of high accuracy. An interesting detail of

C&P is that, although its algorithms are nonrelativistic, the starting model can be of relativistic accuracy, a fact increasing in turn the overall accuracy of the method.

2 Brief description of the numerical method

The structure and the rotational behavior of the model under consideration are identical to those of Paper II (§ 2). The toroidal magnetic field is defined as in Paper III (§ 2). For clarity and convenience, we shall use the definitions and symbols of Papers II and III. An extensive set of parameters involved in the computations is given in Paper III (Table 1). The primary algorithm of CIT is developed and analyzed in Paper I (§ 3). The flowchart of this method is given in Paper V (Fig. 1).

A brief description of C&P, expressed in Fortran 90, is given below. Note that many (rather obvious) details are hidden behind a few driving subprograms.

```
........10........20........30........40........50........60........70....
12345678901234567890123456789012345678901234567890123456789012345678901234
!=========================================
!  The C&P method (written in Fortran 90)
!=========================================
!  Definition of the PPA environment (as in ME85).
!  N_PPA: partition (128 subintervals, say) of [0,DENSITY_UPPER].
      CALL DEFINE_PPA_ENVIRONMENT
!  Definition of the CIT environment for an undistorted
!  configuration with rotational perturbation parameter
!  YVAR=0 and magnetic perturbation parameter HVAR=0.
!  NAC: partition (128 subintervals, say) of [0,1].
!  NRC: partition (256 subintervals, say) of [0,R_MAX].
!++++++++++++++++++++++++++++
 1000 CONTINUE ! The main loop
!++++++++++++++++++++++++++++
      CALL DEFINE_CIT_ENVIRONMENT(YVAR,HVAR)
!  Returns the folowing NAC×NRC arrays:
!  GMAT, the gravitational potential matrix;
!  WMAT, the differential rotation matrix;
!  TMAT, the polytropic density matrix;
!  TORMAT, the toroidal magnetic field matrix;
!  PLI_CIT, the polytropic index matrix;
!  CONSK_CIT, the polytropic pressure constant matrix.
!=======================
!  The primary algorithm
!=======================
      WHERE (TMAT>0)
          UMAT=GMAT+WMAT-TMAT-TORMAT
      ELSEWHERE
          UMAT=0
      END WHERE
      WHERE (ABS(UMAT)>UMAT_TOL)
          NONZERO_LOCATION=1  !  NAC×NRC integer array
      ELSEWHERE
          NONZERO_LOCATION=0
      END WHERE
      U0AVRG=SUM(UMAT)/SUM(NONZERO_LOCATION)
      TMAT_TEMPORARY=GMAT+WMAT-U0AVRG
!  Solve the equation(s):
!  TMAT+HVAR*SMAT**2*TMAT**PLI_CIT/4-TMAT_TEMPORARY=0.
!  SMAT: NAC×NRC array of the corresponding cylindrical coordinate(s).
      CALL SOLVE_TMAT_EQUATION
```

```
........10........20........30........40........50........60........70....
12345678901234567890123456789012345678901234567890123456789012345678901234
!  Check for convergence:
!  |TMAT_NOW-TMAT_PREVIOUS|<TMAT_TOL .AND.
!  |U0AVRG_NOW-U0AVRG_PREVIOUS|<U0AVRG_TOL.
      CALL CHECK_CONVERGENCE(CHECK_FLAG)
      IF (CHECK_FLAG==0) THEN
          GOTO 1000
      ELSE
          CONTINUE
      END IF
!  Update the PLI_CIT and CONSK_CIT matrices:
      CALL DEFINE_CIT_SUB_ENVIRONMENT
!=====================
!  Rescaling the units
!=====================
      CALL RESCALE_UNITS(PHYS_RADIUS,PHYS_MASS,PHYS_GRAV_POT_ENERGY, &
     &                   LENGTH_UNIT,DENSITY_UNIT,ENERGY_UNIT)
!  Where:
!  LENGTH_UNIT=PHYS_RADIUS/DIMENSIONLESS_RADIUS,
!  DENSITY_UNIT=PHYS_MASS/(4*PI*LENGTH_UNIT**3*DIMENSIONLESS_MASS),
!  ENERGY_UNIT=PHYS_GRAV_POT_ENERGY/DIMENSIONLESS_GRAV_POT_ENERGY.
!  Then rescale the derived units:
      CALL RESCALE_DERIVED_UNITS
!  Then compute the physical parameters, i.e.,
!  volume, mass, angular momentum, rotational kinetic energy, etc.
      CALL COMPUTE_PHYSICAL_PARAMETERS
!======================================
!  Computing a distorted configuration
!======================================
!  Set:
      YVAR=YVAR_GIVEN
!  and
      HVAR=HVAR_GIVEN
!  Then go to the top of the main loop:
      GOTO 1000
!=========================
!  End of the main program
      END
!=========================
```

3 Some preliminary tests

C&P has been tested in a somewhat preliminary way. In particular, we have computed (1) rotating white dwarf models similar to those in Paper II (Table 13), (2) rotating white dwarf models similar to those in Hachisu (1986, Tables), (3) rotating magnetic white dwarf models similar to those in Paper III (Tables 2–7), and (4) rotating models with densities intermediate to white dwarfs and neutron stars, similar to those in ME85 (§§ 2–3). The differences in the corresponding results do not exceed 2%.

On the other hand, we have computed neutron star models obeying some typical equations of state. We have made use of the "rescaling the units" advantage discussed above. So, the corresponding nondistorted models have relativistic accuracy. However, as the desired model ratio "polar radius / equatorial radius" decreases, the errors of C&P (in comparison with corresponding relativistic computations) increase appreciably. Nevertheless, we have remarked that the aim of C&P is not to substitute specialized relativistic algorithms, but rather to provide us with an alternative simple way to for roughly computing models with heavy physical input.

Acknowledgments

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