A Scilab PROGRAM FOR COMPUTING ROTATING MAGNETIC COMPACT OBJECTS

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Abstract

We implement the so-called "complex-plane iterative technique" (CIT) to the computation of classical differentially rotating magnetic white dwarf and neutron star models. The program has been written in SCILAB (© INRIA-ENPC), a matrix-oriented high-level programming language, which can be downloaded free of charge from the site `http://www-rocq.inria.fr/scilab`. Due to the advanced capabilities of this language, the code is short and understandable. Highlights of the program are: (a) time-saving character, (b) easy use due to the built-in graphics user interface, (c) easy interfacing with Fortran via online dynamic link. We interpret our numerical results in various ways by extensively using the graphics environment of SCILAB.

1 Introduction

The computation of white dwarf and/or neutron star models was the aim of many investigations so far. Among the large number of techniques developed to solve the problem, the most efficient method to treat strongly distorted configurations is the so-called "self-consistent field method" (SCF), originally developed by Ostriker & Mark (1968). SCF is not a strictly defined method, but rather a computational scheme, which can be implemented in various ways that follow the basic idea. SCF methods are characterized by excellent accuracy, which can be achieved after a relatively small number of iterations (less than one hundred, in most cases). They can be used for determining the equilibrium structure of barotropic or even pseudo-barotropic models, which may be highly distorted by mechanisms such as rotation and/or magnetic fields. Due to these advantages, SCF methods have been used in many investigations so far (see, e.g., Mark 1968; Ostriker & Bodenheimer 1968; Bodenheimer & Ostriker 1970; Jackson 1970; Bodenheimer 1971; Bodenheimer & Ostriker 1973; Bisnovatyi-Kogan & Blinnikov 1974; Clement 1974; Blinnikov 1975; Stahler 1983a,b; Hachisu 1986a; Hachisu 1986b).

An implementation of the SCF method is the so-called "complex-plane iterative technique" (CIT), initially developed for computing visco-polytropic models distorted by rigid or differential rotation (Geroyannis 1991, hereafter Paper I). CIT was further extended for the computation of polytropic or white dwarf models distorted by rotation and/or magnetic fields (Geroyannis & Papasotiriou 1994, hereafter Paper II; Geroyannis & Papasotiriou 1997, hereafter Paper III; Geroyannis & Papasotiriou 1999, hereafter Paper IV; Papasotiriou & Geroyannis 2001).

CIT has been implemented in Fortran 90. The resulting executable code runs remarkably fast on a typical personal computer. However, the program is very large, and, although fully modularized, not so easy to understand. In this paper we present an implementation of CIT written in SCILAB, which is a large mathematical package providing a powerful high level programming language. Among the similar mathematical packages available, we choosed SCILAB because (a) it is free, easily accesible to anyone, (b) it is almost entirely written in Fortran, so many SCILAB commands are similar to those of Fortran, (c) it uses large, well-tested numerical analysis libraries, also available for free, and written in Fortran. By making use of the extensive capabilities of the Scilab programming language, the resulting Scilab implementation of CIT is a short and easily understable program.

2 The model

The model under consideration is a completely degenerated configuration, distorted by rotation and toroidal magnetic field. A detailed description of the model can be found in Papers II and III.

The equation of state is given by (Chandrasekhar 1939, hereafter C39; chap. X, eq. [20]),

$$P = A\left[x\left(2x^2-3\right)\sqrt{x^2+1}+3\mathrm{arcsinh}\left(x\right)\right], \tag{1}$$

where P is the total pressure, x is the threshold momentum, and $A = 6.01 \times 10^{22}\,\mathrm{dynes/cm^2}$ for the white dwarf case or $A = 6.86 \times 10^{35}\,\mathrm{dynes/cm^2}$ for the neutron star case (ideal neutron gas equation of state; Shapiro & Teukolski 1983, § 2.3).

The model of the differential rotation adopted is the generalized Clement's model of differential rotation (see, e.g., Paper IV, eq. [2.3]),

$$\omega\left(s\right) = \sqrt{\sum_{i=1}^{4} a_i e^{-b_i F_r s^2}}, \tag{2}$$

where ω is the differential rotation, s is the dimensionless cylindrical coordinate, a_i and b_i are the so-called non-uniformity parameters, and F_r is the strength of the differential rotation; $F_r = 0$ denotes rigid rotation; $F_r = 1$ denotes the highest possible differential rotation. Furthermore, we decide to introduce two kinds of differential rotation, with respect of the ratio $\Omega_\mathrm{e}/\Omega_\mathrm{c}$, where Ω_c and Ω_e are the values of angular velocity at the center and the equator, respectively:

1. The "variable ratio" model, where the ratio $\Omega_\mathrm{e}/\Omega_\mathrm{c}$ of a distorted model is considered variable, defined by equation (2). The term "variable" denotes that a distorted model has an equatorial radius ξ_e which is usually different from the radius of the corresponding non-distorted model, ξ_1. Therefore, the ratio $\Omega_\mathrm{e}/\Omega_\mathrm{c}$ of the non-distorted model is not equal to the corresponding ratio of the distorted model.

2. The "constant ratio" model, where the ratio $\Omega_\mathrm{e}/\Omega_\mathrm{c}$ is considered constant and equal to the corresponding ratio of the non-distorted model. In this view, equation (2) is valid only for the non-distorted model. The corresponding equation for a distorted model is

$$\omega\left(s\right) = \sqrt{\sum_{i=1}^{4} a_i e^{-b_i F_r \left(\frac{\xi_1}{\xi_\mathrm{e}} s\right)^2}}, \tag{3}$$

   where ξ_1 is the radius of the non-distorted model and ξ_e is the radius of the corresponding distorted model.

It is worth emphasizing that the neutron star models considered here are "classical" models, which can be computed using the Chandrasekhar's theory of completely degenerate configurations. **A new Scilab program for computing general relativistic neutron star models is in preparation.**

3 The numerical method

CIT is an iterative method, aiming basically to the computation of (a) the generalized Lane-Emden function, Φ, and (b) the gravitational potential inside the star, U^{i}, at a predefined set of grid points concerning the dimensionless linear spherical coordinate, ξ, and the modified angular spherical coordinate, $\nu = \cos\theta$. The grid points are given by equations

$$\nu_i = \frac{i-1}{KAP-1},\ i = 1, \ldots, KAP, \tag{4}$$

$$\xi_j = \frac{j-1}{KRP-1}\xi_{\mathrm{up}},\ j = 1, \ldots, KRP, \tag{5}$$

where KAP and KRP are the number of grid points for ν and ξ coordinates, respectively, and ξ_{up} is the upper limit of ξ. In other words, the basic aim of CIT is to compute the matrices $\Phi_{ij} = \Phi\left(\nu_i, \xi_j\right)$ and $U^{\mathrm{i}}_{ij} = U^{\mathrm{i}}\left(\nu_i, \xi_j\right)$.

In brief, the CIT algorithm starts by solving the Lane-Emden differential equation (C39, chap XI, eq. [28]) and thus obtaining an initial guess of the generalized Lane-Emden matrix Φ_{ij}. This guess is used to compute an estimation of the gravitational potential matrix U^{i}_{ij} using eq. [3.17] of Paper I. The gravitational potential matrix U^{i}_{ij} can, in turn, be used to compute a new estimation of Φ_{ij}, using the equation of equilibrium. This new estimation of Φ_{ij} can then be used to compute a new estimation of U^{i}_{ij} and so on, until some termination criteria are satisfied.

A critical limitation concerning the Lane-Emden equation (used to obtain the initail guess of Φ_{ij}) is that it cannot be solved numerically beyond its first root ξ_1. To surpass this problem, we decide to solve the Lane-Emden equation in the complex plane. In Scilab, this can be easily done using its powerful command `ode`, which is able to work in the complex plane. After finishing the numerical solution of the Lane-Emden equation, we take the real part of the solution. This trick permits the computation of the Lane-Emden function far beyond its first root ξ_1, and works surprisingly well.

Roughly speaking, the method converges exponentially. After finishing the iteration process, the final values of Φ_{ij} and U^{i}_{ij} can be used to compute various structure parameters of the model. The reader can find an extensive table of structure parameter definitions at Paper III (Table I). A raw flowchart of CIT can be found in Geroyannis & Papasotiriou (2000, Fig. 1).

4 The Scilab program

Our Scilab program implements the algorithm of CIT. The program is menu-driven, using Scilab's simple graphical user interface commands. After finishing the iterational process, the program computes and (optionally) stores into an ASCII file many structure parameters, both in dimensionless and in cgs form. The program can also display many graphical representations, aiming to closely follow the iterational process and the final results obtained. All these graphics representations are not enabled by default, but the user can enable some or all of them at will, using the graphical user interface.

Our Scilab program consists of the main program, namely `CIT.sce`, and three "libraries", i.e. collections of functions used by the main program: (a) the library `CITfunctions.sci`, which contains functions used by the main algorithm of CIT, (b) the library `CITgraphics.sci`, which contains graphics functions, and (c) the library `CITgui.sci`, which contains functions of the graphical user interface. Main program also calls the object code of a small Fortran library, namely `CITfortran.f`, which is responsible for the computation of the gravitational potential, and for computing double integrals.

The program contains some explanatory remarks and should be easily understable, provided a sufficient knowledge of the SCILAB programming language is achieved. The reader can obtain the source code of the program by contacting the authors (e-mails: `papasot@physics.upatras.gr`, `vgeroyan@physics.upatras.gr`).

5 Discussion

The SCILAB code has been tested thoroughly, by comparing its results with the corresponding results of the large Fortran 90 code. Our tests included a variety of models, covering all possible values of the model parameters. We have found only negligible differences, so the code gives accurate results in any case.

On a typical personal computer, computation time for the SCILAB code varies from some seconds to some minutes, depending on the grid size and the basic model parameter $1/y_0^2$. These computation times are longer than the corresponding computational times for the Fortran code. This is because SCILAB, as every other mathematical package available, is an interpreted language. So, the SCILAB code, although not as fast as the corresponding Fortran 90 code, is sufficiently fast for practical purposes concerning rotating magnetic compact stars.

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